Verification for Adaptive Mesh Refinement Code

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We present the results from verification and convergence analysis of adaptive mesh refinement (AMR) calculations for two different AMR codes. Code verification is extremely important for science-based prediction and simulation. Previous verification efforts focused on the convergence behavior of uniform grid. With AMR, we can obtain more accurate results with substantially less computational cost. The ideal goal of AMR is to *achieve the same accuracy in the refinement region as in the corresponding fine uniform grid*. However, test results shows that AMR may not achieve the convergence of the equivalent finest uniform grid. In some cases, numerical results with AMR have even larger error than those without AMR. AMR can also trigger an instability for some applications.

We have investigated three model problems. The first two have smooth solutions and the third one contains a shock discontinuity. All of them have exact solutions and represent a variety of problems. We have solved the problems with two hydrodynamics AMR packages: patch-based AMR-MHD [3] and cell-based RAGE AMR. In AMR-MHD, we have tested two time-step methods: local step where the time step varies with the refinement level and locked step where the same stepsize is used for all refinement levels.

The first model is a linear wave problem which advects a Gaussian density profile along the diagonal of a rectangular domain. Figure 1 shows that one AMR calculation by AMR-MHD package achieves the same accuracy as the finest uniform grid. Figure 2 shows the performance of RAGE AMR. We see that only a few AMR calculations achieve better performance in L_∞ error.

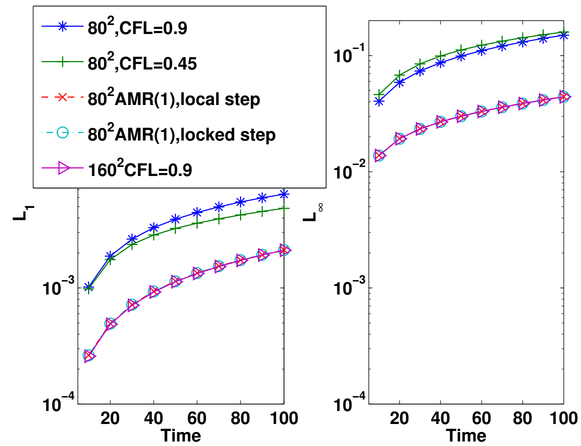

Figure 1: Error vs. Time for AMR-MHD. AMR with local or locked step (80^2AMR(1)) achieves accuracy of the finest resolution grid (160^2). AMR(n) denotes AMR with n-level refinement.

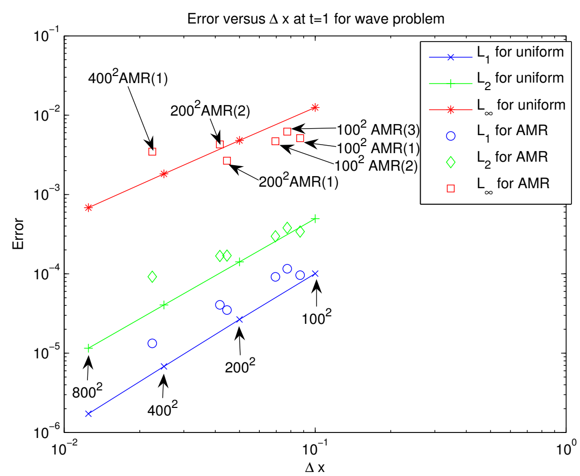

Figure 2: Error vs. local spacing for the RAGE AMR calculation. The same error with a larger Δx means better convergence. Two abnormal things: three-level refinement for 100^2 base grid (shown as 100^2AMR(3)) has larger error than one or two level refinement; 1-level AMR for 400^2 base grid has larger error than without AMR.

The second model problem is a vortex advected along the diagonal of a rectangular domain. It has exactly the same density solution as the linear wave problem. However, the velocity field and pressure are not constant, and the problem becomes essentially a nonlinear problem. Figure 3 shows the impact of nonlinearity through the results of the AMR-MHD package: AMR with locked step has larger error than AMR with the local step, and even has larger error than the coarse grid without AMR after some time. Our results

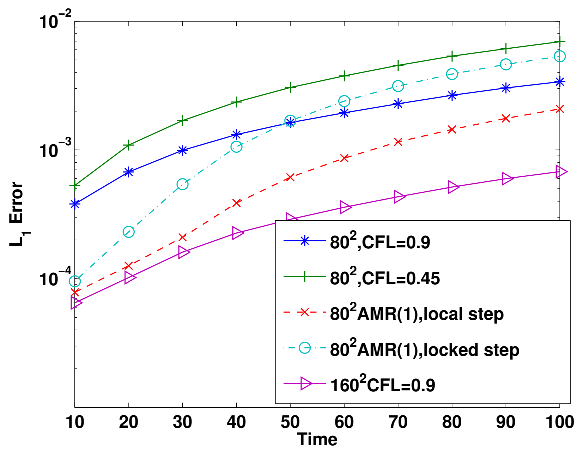

Figure 3: Error vs. Time for vortex problems. AMR with locked step has larger error than with local step. It even has larger error than coarse uniform grid after t=50.

with RAGE AMR for this problem exhibit greater errors than for the linear wave problem due to the nonlinearity of the problem (see [1] for detail).

The third example is Noh's shock tube problem. For planar Noh's problem solved on Cartesian grid, AMR-MHD achieves accuracy of the finest resolution grid, whereas RAGE AMR diverges with more refinement levels. The spherical Noh's problem is solved on (r,z) cylindrical grid where we observed a numerical shock instability, the *carbuncle* phenomenon, in both AMR results with three or more refinement levels (see Figure 4). This anomaly was also observed by Gisler [2].

Our comparison of these two AMR codes on these problems has raised several issues regarding the effectiveness of RAGE AMR code; see [1] for more details. Some issues have already been

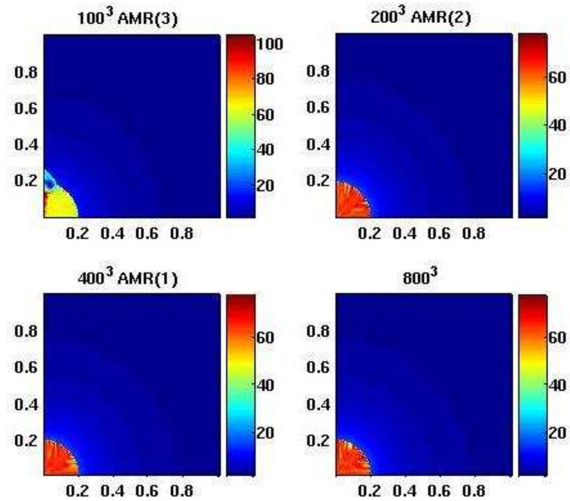

Figure 4: Density plot for different refinement levels of the same finest resolutions for Noh's spherical problem. The three-level refinement has a density bubble straddling the shock near $r=0$.

addressed by the code development team.

We have also found two AMR convergence issues that may occur in a general AMR code: (1) AMR with locked time step has larger dispersion (phase) error than corresponding fine uniform grid. (2) AMR can trigger the *carbuncle* instability at shock front for cylindrical (r,z) coordinate near z-axes. Both issues need further research.

Acknowledgements

Los Alamos Report LA-UR-05-7985. We would like to acknowledge Advanced Simulation and Computing (ASC), verification and validation project, for financial support.

References

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[3] S. Li and H. Li, A Modern Code for Solving Magneto-hydrodynamic or Hydrodynamic Equations, Technical Report , Los Alamos National Lab, 2003.